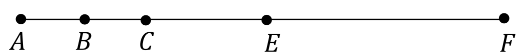
Lesson 1.06 Properties of Lines

Students will be able to:

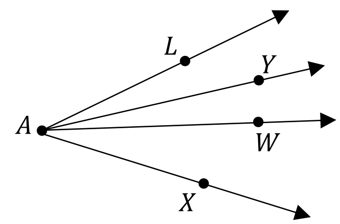
- <u>Content Objective:</u> Explain the difference between a postulate and a theorem.
- <u>Language Objective</u>: Write conclusions based on investigations with points and lines.

Warm Up

1. In the diagram below, E is the midpoint of $\overline{AF}$, C is the midpoint of $\overline{AE}$ and B is the midpoint of $\overline{AC}$. If $AB = 2$, find the length of $\overline{EF}$.

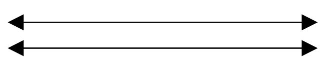
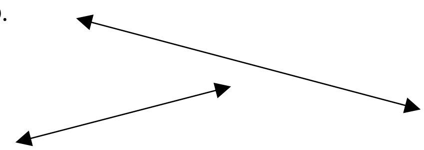
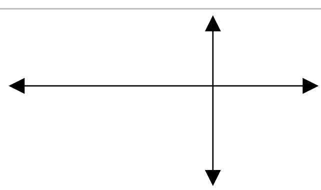
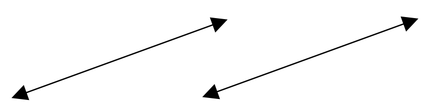


2. In the diagram below, $\overrightarrow{AW}$ is the angle bisector of $\angle LAX$, and $\overrightarrow{AY}$ is the angle bisector of $\angle LAW$. If $m\angle LAX = 42°$ (not drawn to scale), find the measure of $\angle WAY$.



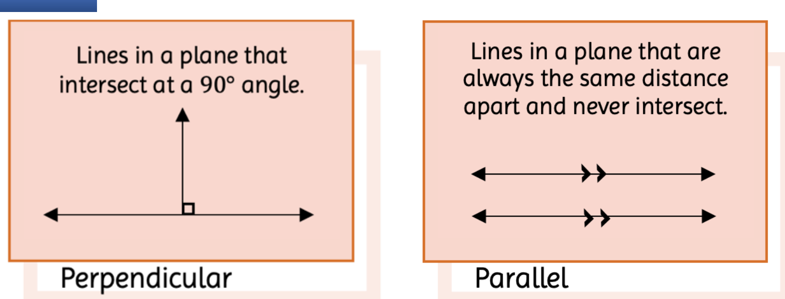
Vocabulary Review

Using your prior knowledge of geometry, identify whether each of the following lines are parallel, perpendicular or neither. Use measurement if necessary.

a. 	b. 
c. 	d. 

Now that you reviewed your general understanding of parallel and perpendicular lines, lets look at these definitions more closely.

Graphic Organizer



Investigate

Is it possible to draw more than one line through points J and K below?

a. Experiment below by constructing lines through points J and K.

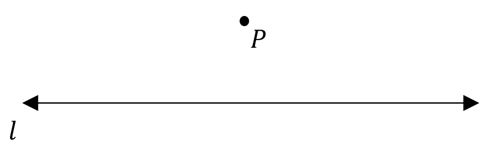
b. What can we conclude based on part a.?

Investigate

Is it possible to draw more than one parallel line through point P, **not on the line l**, <u>parallel</u> to line l ?

a. Experiment below by constructing lines through point P.

a. What can we conclude based on part a.? This is known as **Playfair's Axiom**.

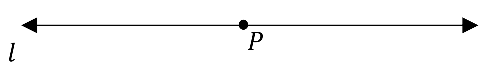


Investigate

Is it possible to draw more than one line through point P **on l**, <u>perpendicular</u> to line l ?

a. Experiment below by constructing lines through point P.

b. What can we conclude based on part a.?

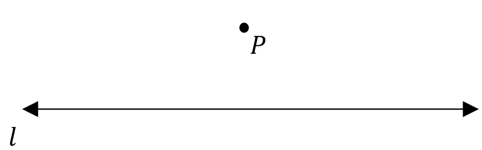


Investigate

Is it possible to draw more than one line through point P ***not* on l**, <u>perpendicular</u> to line l ?

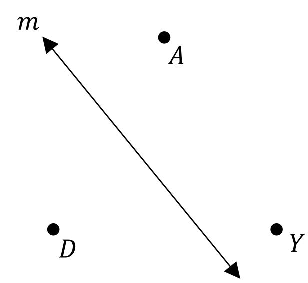
b. Experiment below by constructing lines through point P.

c. What can we conclude based on part a.?



Write It Out

Consider the diagram below where line m divides the plane into two **half planes**.



a. Construct $\overline{AD}$. Is It true that a segment connecting two points on opposite sides of a line m must intersect line m? Explain.

b. Construct $\overline{AY}$. Is it true that a segment connecting two points on the same side of line m will always be parallel to line m? Explain.

c. Construct $\overline{DY}$. Using your prior knowledge of geometry, what type of triangle is ΔADY? Use measurement to justify your answer.

The conclusions we have been making based on the investigations above are known as **postulates** or **axioms**. In Euclid's book "Elements", he introduced 5 postulates that, at that time, the whole of geometry was solely based on.

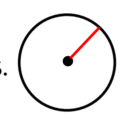
Euclid's Postulates

Mathematical truth statements that are accepted (believed) without having to prove.

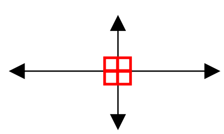
Postulate I: Through any two points, there is exactly one line.

Postulate II: Any line segment can be extended infinitely in both directions.

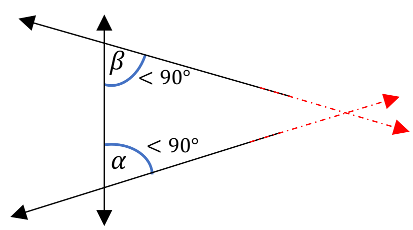
Postulate III: A circle can be described as a center point and a radius.



Postulate IV: All right angles are equal.



Postulate V: <u>(Parallel Postulate)</u>
If two lines cut by a transversal form two same side interior angles less than 90°, then the two lines will intersect on that side.



The first four postulates are intuitive and can be assumed when conducting future proofs in this course. The fifth postulate (parallel postulate) is not so intuitive.

Check Point

Explain the difference between a postulate and a theorem.

Problem Set

Name: ____

1. **True or False.** Identify whether each of the following statements are true or false by writing T or F.

   a. ______ The intersection of two lines is a line.

   b. ______ Parallel lines never intersect.

   c. ______ Through a point not on a given line, there exists at least one line through the point parallel to the given line.

   d. ______ Through a point not on a given line, there exists exactly one line through the point perpendicular to the given line.

   e. ______ An equilateral triangle has at least two congruent sides.

   f. ______ A postulate or axiom is a mathematical statement that can be proven.

   g. ______ A theorem is a mathematical statement that can be assumed and is intuitive.

2. **Fill in the blank**. Fill in the blanks with correct word or letter.

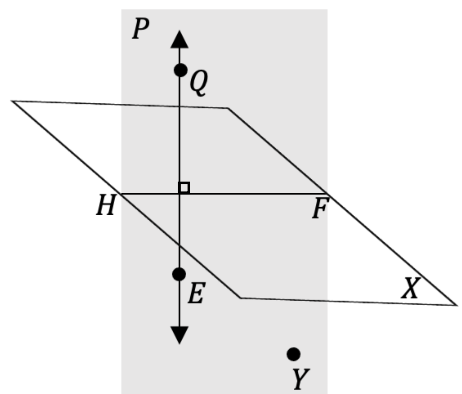
   a. Planes P and X intersect at a __________.

   b. QE is ________________ to HF.

   c. Plane _____ contains $\overleftrightarrow{QE}$.

   d. Planes P and X contain points _____ and _____.

   e. The only line parallel to HF that goes through point F is _________.



3. Euclid's parallel postulate can be rephrased many ways using previous axioms. In Geometry, the converse of a statement can be written by switching the hypothesis and conclusion. For example:

<u>Parallel postulate:</u>	<u>Converse of Parallel Postulate:</u>
If two lines cut by a transversal form two same side interior angles whose sum is equivalent to two right angles, then the two lines will not intersect and are parallel.	Two straight lines intersected by a transversal are parallel if the interior angles formed have a sum equivalent to the sum of two right angles.

Based on the above and the diagram shown, are lines m and n parallel? Use measurement to justify your answer.