**Lesson 2.02 Rational Exponents**

Students will be able to:

- Content Objective: Evaluate an expression in an equivalent form using rational exponents.
- Language Objective: Explain the steps for rewriting rational exponents as roots.

## Warm Up

Simplify each of the following using laws of exponents.

a. $(-2a^2b^3)^{-2} \cdot 8a^3$

b. $\left(\frac{10x^{-4}y^5}{5y^2}\right)^2$

## Vocabulary Review

**Matching-** Match each of the following rules of exponents its corresponding example.

1. ______ Power to a Power
2. ______ Quotient Rule
3. ______ Extended Power Rule (Division)
4. ______ Product Rule
5. ______ Extended Power Rule (Multiplication)

a. $\left(\frac{x}{y}\right)^b$

b. $(xy)^b$

c. $z^x \cdot z^y$

d. $(x^a)^b$

e. $\frac{z^x}{z^y}$

## Graphic Organizer

**Rational Exponents**

| **Definition** | **Notation** |
| --- | --- |
| A rational exponent is a fractional exponent | $b^{\frac{m}{n}} = \sqrt[n]{b^m}$ |
| **Example** | **Example** |
| $2^{\frac{6}{5}} = \sqrt[5]{2^6}$ | $\sqrt[4]{5^3} = 5^{\frac{3}{4}}$ |

## Investigate

1. Evaluate each of the following using a calculator.

a. $25^{\frac{1}{2}} =$ b. $64^{\frac{1}{2}} =$ c. $343^{\frac{1}{3}} =$

2. What do you notice? What can we say about the denominator of fractional exponents?

## Skill 1: Writing Roots

Rewrite each of the following as roots instead of fractional exponents. Then evaluate the expression.

a. $125^{\frac{1}{3}}$ b. $4^{\frac{3}{2}}$

c. $9^{-\frac{1}{2}}$ d. $x^{-\frac{1}{3}}$

## Exercise 1: Writing Roots

Rewrite each of the following as roots instead of fractional exponents. Then evaluate the expression.

a. $25^{\frac{3}{2}}$ b. $16^{\frac{1}{4}}$

c. $32^{-\frac{1}{5}}$ d. $x^{-\frac{2}{3}}$

## Skill 2: Exponential Form

Rewrite in exponential form, then simplify completely.

a. $\sqrt{81}$ b. $\left(\sqrt[3]{4}\right)^6$ c. $\sqrt[4]{x^8}$

d. Solve the equation $9 + 5\sqrt[3]{2x} = 29$ for $x$ using your knowledge of rational exponents and roots.

## Exercise 2: Exponential Form

Rewrite in exponential form, then simplify completely.

a. $\sqrt{49}$

b. $\sqrt[3]{125^2}$

c. $\sqrt{x^5}$

d. Solve the equation $7 = y^{\frac{1}{2}}$ for $y$ using your knowledge of rational exponents and roots.

## Write It Out

Explain how $\left(2^{\frac{1}{5}}\right)^3$ can be written as the equivalent radical expression $\sqrt[5]{8}$.

## Skill 3: Variables

When $a > 0$ and $b$ is a positive integer, the expression $(2a)^{\frac{3}{b}}$ is equivalent to

(1) $\frac{1}{\left(\sqrt[b]{2a}\right)^3}$

(2) $\left(\sqrt[3]{2a}\right)^b$

(3) $\frac{1}{\sqrt[3]{2a^b}}$

(4) $\left(\sqrt[b]{2a}\right)^3$

## Exercise 3: Variables

Given $y > 0$, the expression $\sqrt{3x^2y} \cdot \sqrt[3]{27x^3y}$ is equivalent to

(1) $3^{1.5}x^2y^{0.2}$

(2) $3^{\frac{3}{2}}x^2y^{\frac{5}{6}}$

(3) $81x^{\frac{5}{6}}y^5$

(4) $3^{\frac{4}{3}}xy$

## Check Point

**Multiple Choice**

For all positive values of $x$, which expression is equivalent to $x^{\frac{3}{5}}$?

(1) $\sqrt[3]{x^5}$

(2) $(x^3)^5$

(3) $\sqrt[5]{x^3}$

(4) $3(x^5)$

2.02- Problem Set

Name: ____________________________________________

1. **Multiple Choice**
   Which of the following represents $10^{\frac{8}{5}}$ in radical form?
   a. $\left(\sqrt[5]{10}\right)^8$
   b. $\left(\sqrt[10]{8}\right)^5$
   c. $\left(\sqrt[10]{5}\right)^8$
   d. $\left(\sqrt[8]{10}\right)^5$

2. **Multiple Choice**
   Which of the following represents $(5x)^{\frac{5}{2}}$ in radical form?
   a. $\left(\sqrt{5x}\right)^2$
   b. $\left(\sqrt[5]{5x}\right)^2$
   c. $\left(\sqrt{5x}\right)^5$
   d. $\sqrt{5x}$

3. Rewrite each of the following as roots instead of fractional exponents. Then evaluate the expression.

   a. $5^{\frac{5}{4}}$
   b. $2^{\frac{4}{5}}$
   c. $(10x)^{\frac{3}{5}}$

4. Rewrite in exponential form, then simplify completely.

   a. $\sqrt{5}$
   b. $\sqrt[3]{7^5}$
   c. $\left(\sqrt[3]{6k}\right)^2$