Lesson 2.08 Basic Rigid Motion Proofs
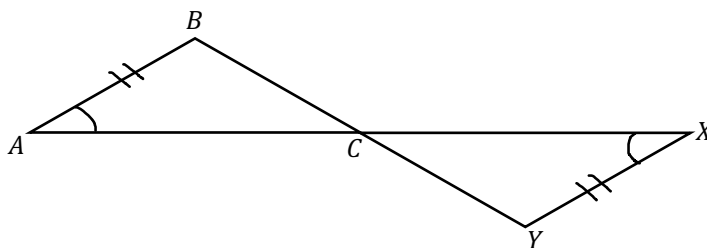
Students will be able to:

- <u>Content Objective:</u> prove that vertical angles are congruent, alternate interior angles are congruent, and corresponding angles are congruent.

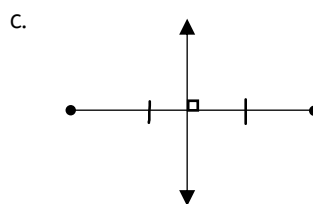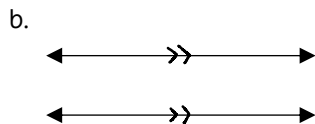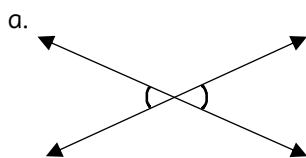- <u>Language Objective</u>: Write organized statements and reasons to construct a geometric proof.

Warm Up

Describe a single rigid motion that maps ΔABC to ΔXYC.

Vocabulary Review

1. Identify each of the geometric concepts being represented below using the word bank.

   a.

   b.

   c.

Word Bank

Vertical Angles

Parallel Lines

Perpendicular Bisector

2. **Fill in the blank.**

   - <u>Rigid Motion:</u> A transformation that maps lines to _____, segments to _____, and rays to _____ and also preserves _____ and _____ measure.

   - <u>Geometric Proof:</u> A collection of true _____ and _____, listed in proper order, that prove mathematical concepts.

In this lesson, we will use rigid motions to prove mathematical concepts we know to be true. First, let's review some important implications of rigid motions and rotations.

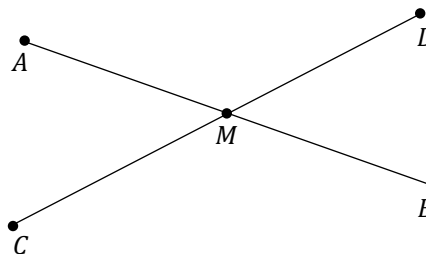<u>Rigid Motions & Rotations</u>

- Rotating a line 180° about a point on the line produces the same line.

- Rotating a line 180° about a point not on the line produces a parallel line.

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Skill 1: Vertical Angles

Segments AB and CD intersect at point M, where M is the midpoint of both line segments.

a. Are $\angle AMC$ and $\angle BMD$ congruent? Use tracing to verify.

b. What rigid motion maps $\angle AMC$ to $\angle BMD$? Be specific.

c. Prove $\angle AMC \cong \angle BMD$.

    1. **Statement:** $\overline{AM}$ gets mapped to $\overline{BM}$ and $\overline{CM}$ gets mapped to $\overline{DM}$ by a rotation of 180° in either direction about point M.

    Reason: _____.

    2. **Statement:** $\angle AMC \cong \angle BMD$

    Reason: _____.

Now that we have proved vertical angles are congruent, let's apply this fact.

Exercise 1: Vertical Angles

$\overleftrightarrow{XY}$ and $\overleftrightarrow{AB}$ intersect at point Z. $m\angle XZA = 3x + 10$, and $m\angle BZY = 7x$. What is the value of x?

1) $\frac{4}{10}$

2) $2\frac{1}{2}$

3) 2

4) $2\frac{1}{4}$

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### Rigid Motions & Translations

- Translations map lines to parallel lines.
- A translation of a line not along the line results in a parallel line.

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Next, we will look at proofs of congruent angles formed by parallel lines cut by a transversal.

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Skill 2: Parallel Lines

Lines l and p are parallel and intersected by transversal t at points B and F respectively.
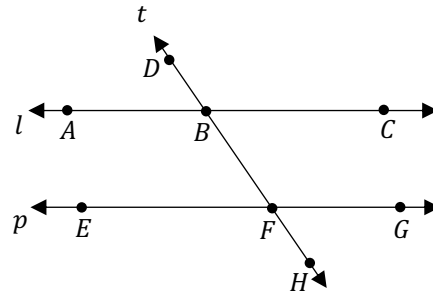
a. $\angle ABF$ and $\angle GFB$ are **alternate interior angles**. Are they congruent? Use tracing paper to verify.

b. Label the midpoint of $\overline{BF}$ point M on the diagram. What single rigid motion would map $\angle ABF$ onto $\angle GFB$?

c. Prove that $\angle ABF \cong \angle GFB$.

   1. **Statement:** $\overrightarrow{AB}$ gets mapped to $\overrightarrow{FG}$ and $\overline{BF}$ gets mapped to $\overline{FB}$ by a rotation of 180° in either direction about point M.

      Reason:

   2. **Statement:** $\angle ABF \cong \angle GFB$

      Reason:

d. $\angle ABF$ and $\angle EFH$ are **corresponding angles.** Are they congruent? Use tracing paper to verify.

e. What single rigid motion would map $\angle ABF$ onto $\angle EFH$?

f. Prove that $\angle ABF \cong \angle EFH$.

1. **Statement:** If $\overleftrightarrow{AB}$ is translated in the direction of $\overrightarrow{BF}$ such that point B gets mapped to point F,then $\overleftrightarrow{AB}$ will map onto $\overleftrightarrow{EF}$.

      Reason:

2. **Statement:** If $\overrightarrow{BF}$ is translated in the direction of $\overrightarrow{BF}$ such that point B gets mapped to point F, then $\overrightarrow{BF}$ will map onto $\overrightarrow{FH}$.

      Reason:

3. **Statement:** $\angle ABF \cong \angle EFH$

      Reason:

Exercise 2: Parallel Lines

Two lines are cute by a transversal. Which condition would always result in the two lines being parallel?

1) Corresponding angles are supplementary.   2) Vertical angles are congruent.

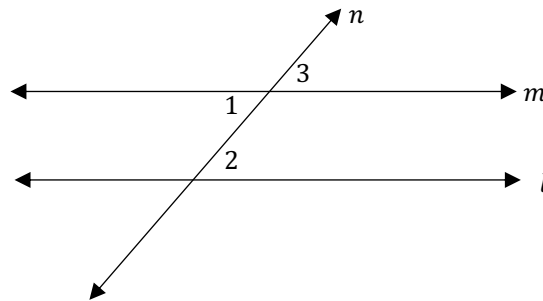3) Alternate interior angles are congruent.   4) Same side interior angles are complementary.

2.08– Problem Set

Name: _____

1. Check all angles that are congruent.

    _____ vertical angles

    _____ supplementary angles

    _____ alternate interior angles

    _____ complementary angles

    _____ corresponding Angles

2. In the diagram below, line n intersects line m and line l.

    a. If the $m\angle 1 = 8x$ and $m\angle 2 = 5x + 12$, what value of x will make $m \parallel n$?

    b. What is the measure of $\angle 3$? Justify your answer.

3. Given parallel lines m and n cut by transversal l.

    a. List two pairs of vertical angles

    b. List two pairs of alternate interior angles.

    c. List two pairs of corresponding angles.

    d. Angles 1 and 8 are alternate exterior angles. What is another pair of alternate exterior angles?