DigitLit math instruction

Lesson 2.08 Modeling Exponential Functions

Students will be able to:

- Content Objective: Express exponential functions using different growth rates (monthly, weekly, daily) in terms of years.
- Language Objective: Explain how laws of exponents aids in manipulating exponential equations and expressions.

Warm Up

Lisa won $3,000 and invested it into an account with an annual interest rate of 3.4%. If her investment were compounded monthly, which expression best represents the value of her investment after t years?

(1) $y = 3000(1.003)^{12t}$

(2) $y = 3000(1.034)^{t}$

(3) $y = \frac{3000(1.034)^t}{12}$

(4) $y = 3009^{12t}$

Graphic Organizer

Different Rates in Terms of Years

- **Yearly rate in terms of years, t:** $y = A(1 \pm r)^t$
- **Monthly rate in terms of years, t:** $y = A\left((1 \pm r)^{\frac{1}{12}}\right)^{12t}$
- **Weekly rate in terms of years, t:** $y = A\left((1 \pm r)^{\frac{1}{52}}\right)^{52t}$
- **Daily rate in terms of years, t:** $y = \left((1 \pm r)^{\frac{1}{365}}\right)^{365t}$

These 4 equations are equivalent.

Use when a question asks you for an equivalent equation/expression

Different Rates in Terms of Different measurements of Time

- **Monthly rate, in terms of months, m:** $y = A\left((1 \pm r)^{\frac{1}{12}}\right)^{m}$
- **Weekly rate, in terms of weeks, w:** $y = A\left((1 \pm r)^{\frac{1}{52}}\right)^{w}$
- **Daily rate, in terms of days, d:** $y = A\left((1 \pm r)^{\frac{1}{365}}\right)^{d}$

Skill 1: Monthly Rate in Terms of Years

Destiny deposits $2500 into a savings account that earns 3% interest per year. The exponential function that models this savings account is $y = 2500(1.03)^t$, where t is the time in years. Write an equation that represents the amount of money in her savings account in terms of the monthly growth rate.

This question is asking for the equation that represents __________interest in terms of ___________.

Exercise 1: Monthly Rate in Terms of Years

The industry that attracts the most inward investment in Ecuador is mines and quarries (about 44%). Edvin decides to invest $5,000 in mine stocks with an annual growth rate of 10%.

a. Write an equation that models the total amount of Edvin's investment, $E(t)$, after one year, t.

b. Write an equation that represents the monthly growth rate of Edvin's investment in terms of years.

Skill 2: Weekly Rate in Terms of Years

Kieran puts $300 into a savings account that earns 3.02% interest annually. The amount in his account can be modeled by $K(t) = 300(1.0302)^t$ where t is the time in years. Write an equation that best approximates the amount of money in his account using a weekly growth rate? Your answer should be of the form $K(t) = 300(b)^{kt}$, where b is rounded to the *nearest thousandth* and k is a whole number.

Exercise 2: Weekly Rate in Terms of Years

Stacy puts $850 into a savings account that earns 5% annually. The amount in her account can be modeled by $S(t) = 850(1.05)^t$ where t is the time in years. Which expression best approximates the amount of money in her account using a weekly growth rate?

(1) $850(1.02019387)^t$

(2) $850(1.00094)^{52t}$

(3) $850(1.02019387)^t$

(4) $26{,}000(1.00094)^{52t}$

Skill 3: Daily Rate in Terms of Years

On average, students who graduated from college in 2010 could compute their growing student loan debt using the function $D(t) = 26{,}000(1.075)^t$, where t is the time in years. Which expression is equivalent to $26{,}000(1.075)^t$ and can be used by students to identify an approximate daily interest rate on their loans.

Based on the options below, this question is asking for the expression that represents _________ interest in terms of ___________.

(1) $26{,}000\left(1.075^{\frac{1}{365}}\right)^t$

(2) $26{,}000\left(\frac{1.075}{365}\right)^{365t}$

(3) $26{,}000\left(1 + \frac{0.075}{365}\right)^t$

(4) $26{,}000\left(1.075^{\frac{1}{365}}\right)^{365t}$

Exercise 3: Daily Rate in Terms of Years

Nick took out a loan for \$590,000 with an annual interest rate of 4.375%. If he makes no payments on this loan, the total amount he will owe, $N(t)$, after t years can be represented by $N(t) = 590{,}000(1.04375)^t$.

a. Which of the can be used to approximate the daily interest rate on this loan?

(1) $590{,}000(1.00011732)^t$

(2) $590{,}000(0.002859589)^{365t}$

(3) $590{,}000(1.000119863)^t$

(4) $590{,}000(1.00011732)^{365t}$

b. What is the daily interest rate based on your answer to part a.?

Write It Out

Explain in words which property of exponents allows for the expressions below to be equal.

$$500\left(1.078^{\frac{1}{12}}\right)^{12t} = 500(1.078)^t$$

2.08- Problem Set

Name: ____

1. Let t = time in years
   m = time in months

Adjust the yearly growth of 6% to show:

a. 6% over 1 year.

b. Monthly rate, expressed in months.

c. Monthly rate, expressed in years.

d. Yearly rate, expressed in months.

2. The growth of a $620 investment can be modeled by the function $P(t) = 620(1.04)^t$, where t represents time in years. In terms of the monthly rate of growth, the value of the investment can be best approximated by

(1) $620(1.00327374)^t$

(2) $620(1.04)^{12t}$

(3) $620(1.00327374)^{12t}$

(4) $590{,}000(1.7647245)^{12t}$

3. The amount of medicine in the bloodstream $M(t)$, that remains after t days can be modeled by the equation $M(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{0.702}}$, where A_0 represents the initial amount of the medication in the bloodstream. An equivalent form of this equation is

(1) $A(t) = A_0\left(\frac{1}{2}\right)^t$

(2) $A(t) = A_0(0.372548091)^t$

(3) $A(t) = A_0(0.3818587827)^t$

(4) $A(t) = A_0(2)^t$