Lesson 3.01 Introduction to Logarithms

Students will be able to:

- Content Objective: Explore the relationship between exponential and logarithmic form and evaluate logarithms.
- Language Objective: Explain how to find the average rate of change using logarithms.

Warm Up

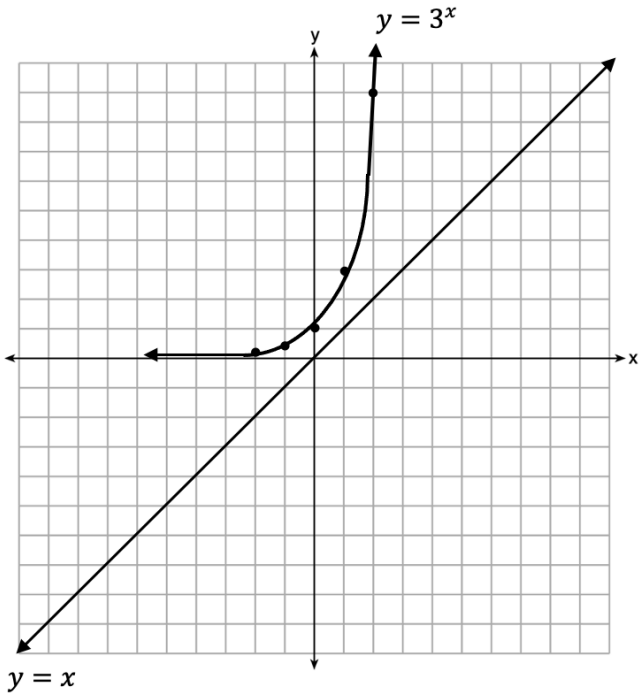
Solve the exponential equation below for the value of x. Hint: Re-write the equation so that both sides have a common base.

$$36^{-2x+3} = \left(\frac{1}{6}\right)^{2x}$$

Investigate

The function $f(x) = 3^x$ is shown graphed on the axes below along with its table of values.

x	-2	-1	0	1	2
$f(x)$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9



a. Is this function one-to-one? What does this tell us about the inverse of this function?

b. Fill in the table of values below for $f^{-1}(x)$. Graph the inverse.

x					
$f^{-1}(x)$					

c. Find the equation for $f^{-1}(x)$. What do you notice?

Graphic Organizer

A logarithmic function is the **inverse** of an exponential function

$$y = b^x \longleftrightarrow y = \log_b x$$

where b is a positive # greater than 1

Definition:

A quantity representing the power to which a fixed number (base) must be raised to produce a given number.

Notation:

$$\log_b a = c \Longleftrightarrow b^c = a$$

b = base

c = exponent

a = argument (result)

Logarithm

Example:

Write $\log_3 243 = 5$ in exponential form

Example:

Write $5^2 = 25$ in logarithmic form

Skill 1: Exponential Form

Convert the following into exponential form.

a. $\log_9 729 = 3$

b. $\log_6 \left(\frac{1}{36}\right) = -2$

Exercise 1: Exponential Form

Convert the following into exponential form.

a. $\log_{13} 169 = 2$

b. $\log_5 \left(\frac{1}{125}\right) = -3$

Skill 2: Logarithmic Form

Convert the following exponential forms to logarithmic form.

a. $7^2 = 49$

b. $5^4 = 625$

c. $2^{-3} = \frac{1}{8}$

Exercise 2: Logarithmic Form

Convert the following exponential forms to logarithmic form.

a. $9^2 = 81$

b. $2^6 = 64$

c. $3^{-3} = \frac{1}{27}$

Skill 3: Evaluate Logarithms

Evaluate each of the following logarithms using exponential form.

a. $\log_2 512$

b. $\log_2\left(\frac{1}{4}\right)$

c. $\log_3 1$

Exercise 3: Evaluate Logarithms

Evaluate each of the following logarithm using exponential form.

a. $\log_3\left(\frac{1}{9}\right)$

b. $\log_{16} 32$

c. $\log_5 5$

Skill 4: Common Logarithm

Calculate each of the following logarithms.

The Common Logarithm
A logarithm with a base of ten. This is a "default" and the 10 is never written, kind of like square roots!

a. $\log 100$

b. $\log 10$

c. $\log (0.01)$

Exercise 4: Common Logarithm

Calculate each of the following logarithms.

a. $\log 100{,}000$

b. $\log (0.0001)$

c. $\log\left(\frac{1}{10}\right)$

Talk it Out

Lisa purchased a new car in May of 2010. The value of her car, V, decreases after t years since 2010 and can be modeled by the function $\log_{0.7} \frac{V}{22000} = t$. What is the average decreasing rate of change per year of the value of the car from May 2011 to May 2013, to the nearest ten dollars per year?

Check Point

Multiple Choice: If $f(x) = b^x$ where $b > 1$, then the inverse of the function is

(1) $f^{-1}(x) = \log_x b$

(2) $f^{-1}(x) = b \log x$

(3) $f^{-1}(x) = \log_b x$

(4) $f^{-1}(x) = x \log b$

3.01- Problem Set

Name: _____

1. Write the following in exponential form.

a. $\log_4\left(\frac{1}{64}\right) = -3$

b. $\log_5\left(\frac{1}{25}\right) = -2$

2. Write the following in logarithmic form.

a. $2^{-5} = \frac{1}{32}$

b. $10^3 = 1000$

3. Evaluate each of the following:

a. $\log_3\left(\frac{1}{9}\right)$

b. $\log_2\left(\frac{1}{64}\right)$

c. $\log_3 27$

d. $\log_{16} 4096$

e. $\log_{625} 125$

f. $\log_{512} 16$

Multiple Choice

4. The value of $\log_m m^{2n}$ is.

(1) n (2) mn

(3) n^2 (4) $2n$

5. Which of the following is the value of $\log_2 \sqrt[3]{64}$?

(1) 2 (2) 8

(3) 3 (4) 16