Lesson 3.04 Doubling Time & Half Life

Students will be able to:

- Content Objective: Solve exponential equations with time adjustments.
- Language Objective: Write an explanation explaining the representation of certain parameters of an exponential equation.

Warm Up

A student studying Statistics created a model for the population of Columbus, where the population decreased 17% over a decade. She used the model $P = 1021(0.83)^d$, where P is the population, in thousands, d decades after the year 2000. Another student, Harold, wants to use a model that would predict the population after y years. Harold's model is best represented by

(1) $P = 1021(0.98)^d$

(2) $P = 1021(0.83)^d$

(3) $P = 1021(0.16)^d$

(4) $P = 1021(8.3)^d$

Vocabulary Review

Researchers are studying a population of 100 penguins. They found that the population grows at a rate of 5% per year. If t is time in years and m is the time in months, match the correct equation to the given information in terms of the context of the problem.

1. ______ 5% over a year
2. ______ Monthly rate, expressed in months
3. ______ Monthly rate, expressed in years
4. ______Yearly rate expressed in months

a. $P = 100(1.05)^t$

b. $P = 100\left(1.05^{\frac{1}{12}}\right)^{12t}$

c. $P = 100(1.05)^{\frac{t}{12}}$

d. $P = 100\left(1.05^{\frac{1}{12}}\right)^{m}$

Skill 1: Doubling Time

When observed by researchers under a microscope, a cellphone screen contained approximately 8,000 bacteria per square inch. Under normal conditions, it is known that bacteria double in population every 20 minutes.

a. Assuming an initial value of 8,000 bacteria, write a function, $b(t)$, that can be used to model the population of bacteria, b, on a cellphone screen, where t represents the time in minutes after it is first observed under a microscope.

b. Using $b(t)$ from part a., determine algebraically, to the nearest hundredth of a minute, the amount of time it would take for a cellphone screen that was not touched or cleaned to have a population of 500,000 bacteria per square inch.

Exercise 1: Doubling Time

Scientists observed a sample containing approximately 500 organisms. The population doubles every 10 days.

a. Write a function $O(d)$, that can be used to model the population of organisms O, where d represents the time in days after it is first observed.

b. Using $O(d)$ from part a., determine algebraically, to the nearest hundredth of a day, the amount of time it would take for the sample of organisms to have a population of 37,000.

Skill 2: Half-Life

The half-life of a radioactive substance is 12 years.

a. Write an equation that can be used to determine the amount, $r(t)$, of 300 grams of this substance that remains after t years.

b. Determine algebraically how long it will take for $\frac{1}{4}$ of this substance to remain.

Exercise 2: Half-Life

Let's revisit exercise #2 from unit 2 lesson 9.

Titrum, a radioactive isotope of hydrogen, has a half-life of 144 months.

a. If a laboratory experiment begins with 100 grams of Titrum, write an equation that represents the number of grams A, of Titrum present after t months.

b. To the nearest hundredth, algebraically determine the number of months, t, it would take for the amount of Titrum to reach 87 grams.

Write It Out

The breakdown of a sample of a certain chemical compound is represented by the function $c(t) = 500(0.5)^t$, where $c(t)$ represents the number of milligrams of the substance and t represents the time in years. In the function $c(t)$, explain what 0.5 and 500 represent in terms of the context of the problem.

Check Point

Solve the equation below for x. Round your answer to the nearest tenth.

$$1800 = 350(2)^{\frac{x}{4}}$$

3.04- Problem Set

Name: ______________________________

1. The half-life of a substance is 15 months. Write an equation that can be used to determine the amount, $s(m)$, of 800 milligrams of this substance that remains after m months.

2. Solve the equation below for x. Round to the nearest hundredth.

$$3\left(\frac{1}{2}\right)^{\frac{x}{2.1}} - 4 = 6005$$

3. A biologist is modeling the population of birds on a tropical island. At the start of the observation, there are 122 birds. The biologist believes that the bird population is doubling every 10 years.

    a. Write a function that models the population of birds, $b(t)$, after t years.

    b. To the nearest year, algebraically determine the number of years it will take for the population of birds to reach 1000.