Lesson 3.05 Solving Exponential Equations using the Natural Logarithm

Students will be able to:

- ▪ <u>Content Objective:</u> Model and solve exponential equations using the natural logarithm.

- ▪ <u>Language Objective:</u> Explain how to use the natural logarithm to solve real life examples.

Warm Up

If $f(t) = 250 \left(\frac{1}{2}\right)^{\frac{t}{4352}}$ represents a mass, in grams, of a certain type of carbon remaining after t years, which statement(s) must be true?

I. The mass of carbon is decreasing by half each year.

II. The mass of the original sample is 250 grams.

(1) I, only   (3) I and II
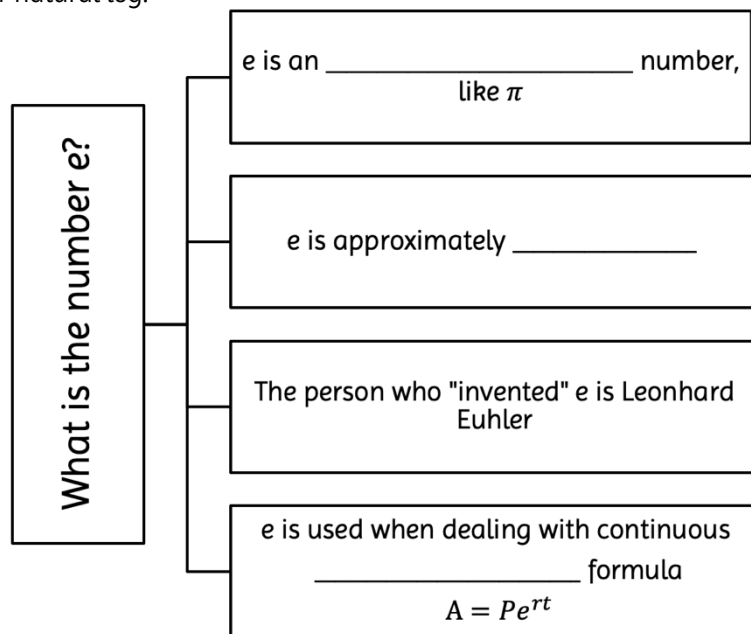
(2) II, only   (4) neither I nor II

Vocabulary Review

Fill in the blank below with the term that best fits the description.

- A _____ logarithm is a logarithm with base 10. It is denoted by $log\,10$ or simply by _____.

- A _____ logarithm is a logarithm with base _____. It can be denoted by log_e but we denote it by _____. This is an abbreviation for *logarithmus naturali*, the Latin translation for natural log.

What is the number e?

- e is an _____ number, like π

- e is approximately _____

- The person who "invented" e is Leonhard Euhler

- e is used when dealing with continuous _____ formula
$$A = Pe^{rt}$$

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Graphic Organizer

The natural log, $\ln x$, is exactly equivalent to $\log_e x$. Any rules for general logs will work for the natural log.

$$\log_e x \qquad \ln x$$
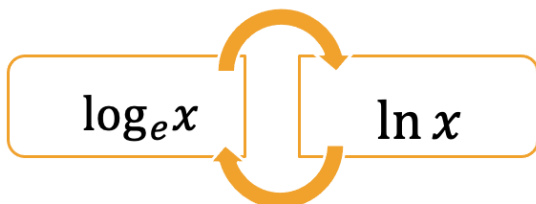
Common Log (base ___)	Natural Log (base ___)
$\log 10 =$	$\ln e =$
$\log 10^5 =$	$\ln e^5 =$
$\log 10^{-3} =$	$\ln e^{-3} =$
$\log 10^x =$	$\ln e^x =$

Skill 1: Solve with ln

Solve for the value of x. Round to the nearest tenth if necessary.

a. $5e^{2x} = 15$

b. $0 = 2e^{2x} - 32$

Exercise 1: Solve with ln

Solve for the value of x. Round to the nearest tenth if necessary.

a. $e^{x-5} = 18$

b. $-2e^{x+10} + 9 = -74$

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Skill 2: Multiple Choice

The solution to the equation $4e^{x+1} = 6$ is

(1) $-1 + ln\left(\frac{3}{2}\right)$

(3) $\frac{3}{2}$

(2) $\left(\frac{ln\ 3}{ln\ 2}\right) - 1$

(4) $-1 + ln\ (1)$

Exercise 2: Multiple Choice

A local community college has a current enrollment of 10,000 students. The enrollment is increasing continuously at a rate of 2.6% each year. Which logarithm is equal to the number of years it will take for the population to increase to 14,000 students?

(1) $\frac{ln(1.4)}{0.26}$

(3) $\frac{ln(1.4)}{2.6}$

(2) $\frac{ln(4000)}{0.026}$

(4) $\frac{ln(1.4)}{0.026}$

Write It Out

A video posted on TikTok initially had 150 views as soon as it was posted. The total number of views to date has been increasing exponentially according to the exponential growth function $y = 150e^{0.2t}$, where t represents time measured in days since the video was posted.

How many total days does it take until 4,494 people have viewed this video? Explain how you arrived at your answer.

Check Point

Multiple Choice

1. Which of the following is the value of $ln\ (e)$?

(1) 0

(3) 1

(2) 10

(4) -1

2. Solve the following equation: $e^{-9x} + 5 = 37$

(1) -0.180

(3) -0.385

(2) -0.199

(4) -0.167

3.05- Problem Set

Name: _____

1. Solve the following equations and round your answers to the nearest hundredth if necessary.

   a. $-6e^{4x} = -6$

   b. $e^{x-9} + 1 = 69$

   c. $4e^{8x-5} + 9 = 73$

   d. $7e^{-x-4} - 5 = 51$

2. The half-life of iodine-124 is about 4 days. The percent of the isotope left in the body d days after being introduced is $I = 300 \left(\frac{1}{2}\right)^{\frac{d}{4}}$. When this equation is written in terms of the number e, the base of the natural logarithm, it is equivalent to $I = 300e^{kd}$. What is the approximate value of the constant, k?

   (1) -0.173

   (3) -2.773

   (2) 0.173

   (4) 2.773

3. A population of pigs on an exotic island can be modeled by $P = 800e^{0.045t}$, where t represents the number of years since the pigs were introduced to the island.

   a. How many pigs were initially introduced at $t = 0$?

   b. *Algebraically* determine the number of years it would take for the population to double. Round your answer to the *nearest tenth* of a year.